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Default strategy in pension saving

The case of Slovakia

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Economic analysis 51

Abstract

This paper analyses saving habits of participants in the Slovakia's private pension scheme and discusses optimal default investment strategies. A typical participant in Slovakia's private pension saving scheme invests mainly in conservative assets that deliver low volatility of returns in the short-run, which might not necessarily be optimal from a long-run perspective. Using resampling simulations based on historical asset returns, we find that an optimal default lifecycling strategy consists of initially investing entirely into equities for first half of individual's career, and later switching new contributions to bonds. Not only new, but also current participants could benefit from switching to default saving strategy. After reaching the retirement age, pensioners should choose a programmed withdrawal of their savings, leaving most of them to accumulate further while decreasing the allocation to stocks. To incentivize the participation in default strategies, we introduce guarantees whose goal is to protect investors from extremely adverse outcomes.

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Executive Summary

Private retirement saving in the II. pillar in Slovakia can be characterized by overly conservative saving strategies. Savers who participate in the II. pillar send part of the mandatory pension insurance contribution to their account and invest them according to their preferences. As of 2017, the savings in guaranteed bond funds accounted for around 80 percent of the total. Participants older than 35 are especially conservative, allocating on average less than 20 percent of their savings into stocks. Both international practice and academic research suggest that creating a default savings strategy can increase return on investment in the long-run while maintaining an acceptable level of investment risk. Such a strategy allocates savings between bonds and stocks according to individual's age, protecting savers from under diversified and underperforming portfolios.

Analysis compares predefined default strategies based on their expected return and associated risk. Among the tested strategies were those with a fixed allocation to equities, lifecycling strategies, where the equity share decreases with increasing age, and dynamic lifecycling strategies, where savings are moved to a conservative (bond) fund once the expected return is achieved.

We find that the best risk-return trade-off at a 40-year investment horizon is achieved by an accumulation strategy, where savings are invested entirely to equities in the initial 20-year period. In the second half, all new contributions are allocated to a conservative bond fund, while the equity portion of the portfolio keeps accumulating. Such a strategy appears to deliver the best investment outcome compared to a pure bond strategy, which we use as a benchmark. If, however, an individual wants to be protected against extremely adverse outcomes, an optimal strategy is stepwise, which smooths out the equity share, i.e. it starts with investing 100 percent in equities and then changes this ratio throughout the investment period so that it reaches 20 percent at retirement.

Our results are robust to variations in income profiles, based either on education or gender. The results are also robust to modifications in the simulation setup for asset returns, to persistently lower expected returns and prolonged career. They also do not change meaningfully if an extra dispersion term is added to simulations to account for the imprecisions in estimating the expected return on stocks.

We show that the optimal saving strategy depends on the setup of the pay-out phase. If an individual chooses to annuitize savings at the retirement or to withdraw a lump sum, the allocation of their savings in equities at this point should not be too large. For these choices, stepwise strategies provide protection against equity crashes. We show, however, that an individual could benefit from choosing a programmed withdrawal. An optimal default strategy consists of an accumulation strategy during the saving phase and a programmed withdrawal with annuitizing the savings at very high age.

To incentivize participation in default strategy, guarantees of minimal return can be offered. Price of such guarantees would depend on the choice of guaranteed return and the assets in the individual's investment portfolio. We show that the price of such a guarantee would be low relative to gains in an adverse scenario.

Optimal strategies discussed above apply also to current savers. If a programmed withdrawal with allocation to stocks is available, current savers should be switched into optimal strategy as long as they have 12 or more years until retirement and into a prolonged safe strategy if they are less than 12 years to retirement. If the programmed withdrawal is unavailable, investor should opt for a safe stepwise strategy.

1 Introduction

Introduced in 2005, the II. pillar of pension insurance is designed to ease the pressures of ageing population on the mandatory PAYG I. pillar. Slovakia will be one of the most rapidly ageing countries in EU. Dependency ratio[1] is expected to increase from 21.5 percent to 57 percent between 2017 and 2070. As the number of pensioners relative to the labour force increases significantly, the PAYG pension system (I. pillar) will be difficult to sustain. The II. pillar aims to improve long-term sustainability of the pension system by shifting expenditures from the future to the present while diversifying individual's pension income, as retirement income of Slovak pensioners consists mostly of public transfers and work income and not from capital income common in other countries OECD, (2017).

Participants in II. pillar invest mainly in bonds, leading to risk of inadequate pension. If individual participates in II. pillar, part of mandatory pension insurance will be transferred to his investment account. Large share of savers are passive, investing entirely into guaranteed bond fund. Bonds yield smaller return, therefore long term investment in bonds increases risk of inadequate pensions. By investing in stocks when younger and shifting portfolio to conservative assets with increasing age, investors can eliminate risk of losing savings before retirement due to decrease in prices of equities and risk of inadequate pension.

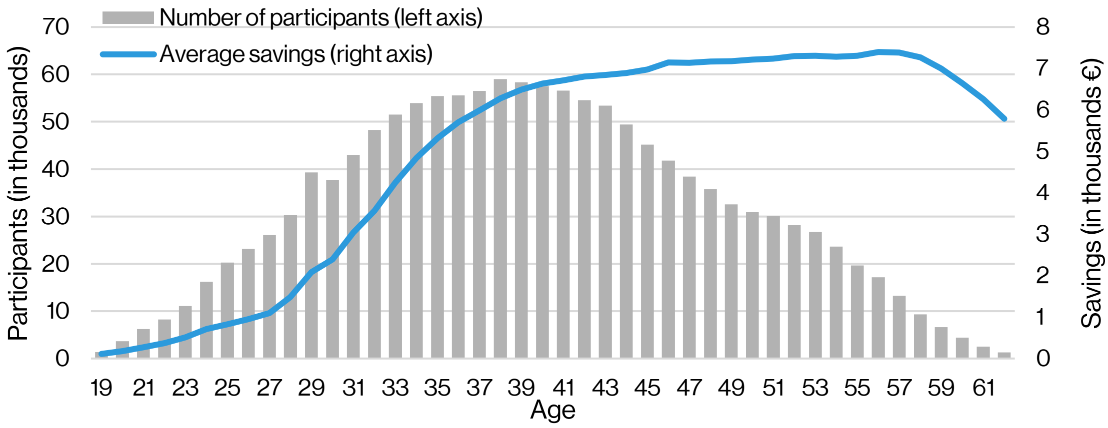
Default strategy can help people optimize retirement savings portfolio. Pension savers are unable or unwilling to make a decision concerning pension saving. Default lifecycling saving strategy defines equity allocation based on investor's age. When choosing default strategy, policy maker needs to consider different income profiles of participants and type of pay-out benefit after reaching retirement. Additionally, optimal saving strategy for current investors needs to be analysed.

[1] Ratio of population 65+ to population 15-64

2 Retirement saving in II. pillar

Nearly one half of the active population participates in II. pillar. Any individual aged 35 or below is eligible to enter but once in, one can no longer opt out. Contribution to pension system is 18 percent of the gross wage. If one chooses to enter the II. pillar, part of the contribution[2] is sent to a personal account at one of the pension management companies (PMC). The transfer from the general system (I. pillar) will then be reduced by the share contributed into the personal scheme (II. pillar).

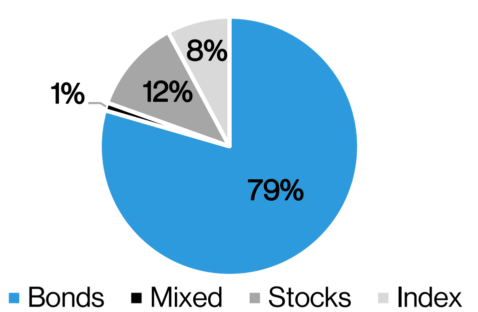
Figure 1: Number of participants (left axis) and average savings (right axis) in according age cohorts (in thousands)



Source: Data provided by PMCs, processed by IFP

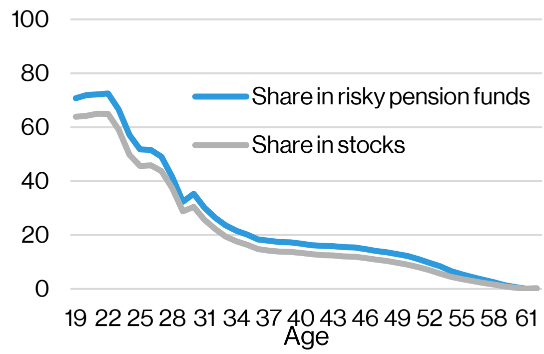
Upon entering the II. pillar, individual chooses the PMC and appropriate investment fund. Each PMC is obliged to operate one guaranteed[3] bond fund and one non-guaranteed stocks fund. Average share of bonds in the bond fund is 88 percent, the rest being money market instruments. The average share of stocks in the stock fund is 66 percent, the rest consists of bonds and money market instruments. Most of the PMCs also offer stock index fund (denoted as index fund) which is linked to chosen stock index fund. Some PMCs also offer mixed fund, but the amount of savings in this fund is very small. One can also split own savings between two funds in which case one of them must be the bond fund. In following, funds containing stocks are denoted risky funds.

Figure 2: Distribution of savings across funds as of 12/2017 (in %)



Source: Data provided by PMCs, processed by IFP

Figure 3: Share of savings in risky assets for age cohorts as of 12/2017 (in % of total savings)



Source: Data provided by PMCs, processed by IFP

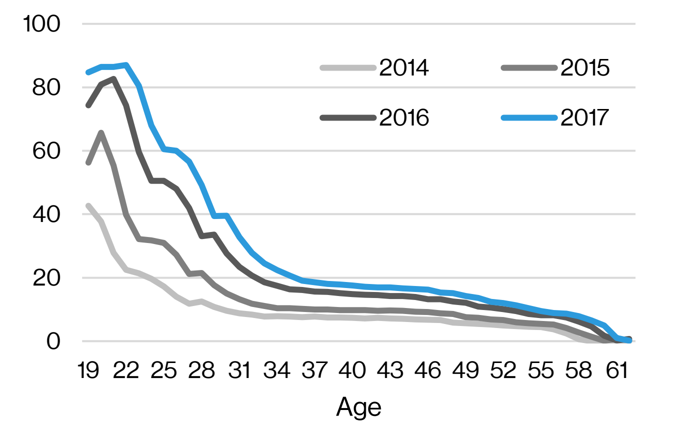
[2] This was 9 % from 2005-2012, then changed to 4 %. From 2017, share increases by 0.25 p.p. until it reaches 6 % where it remains.

[3] The bond fund is the only guaranteed one. The PMC guarantees that the net price of share in this fund will not decrease on 10 year horizon.

Young people tend to invest more aggressively and the share of stocks in investment portfolio decreases with age. Share of stocks in one's portfolio is relatively high for participants under the age of 30. The share of investment to stocks then falls under 20 percent for people above 33 years old. The number of people investing in risky funds increases over time, but this effect is significant only for young people. The share of people aged 30 in risky funds grew from 14 % in 2014 to 42 % in 2017 whereas for people aged 45 it only increased from 9 % to 18 % in the same time span.

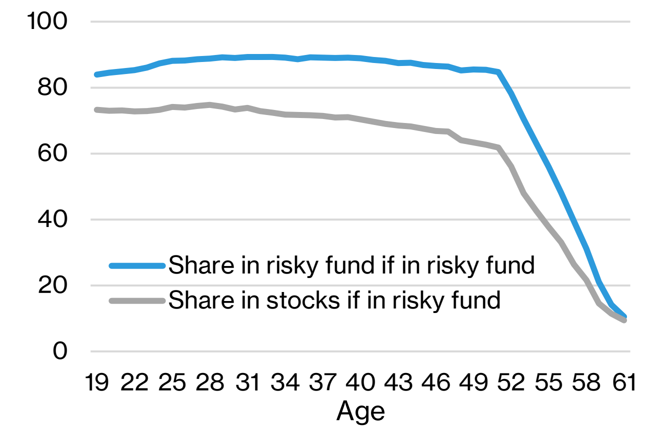
Large share of participants in conservative funds is caused mainly by state's intervention in 2013. Until 2012, the investment strategies were highly regulated[4], providing guarantees on return to participants. As these guarantees were causing inadequate return, they were dropped and participants were automatically transferred to the guaranteed bond fund, with a chance to opt out and remain in non-guaranteed funds. Only small part of participants used this opportunity. This caused the share of participants in risky funds to fall from 88 % in March 2013 to only 9 % in April of the same year. At least[5] 84 % of transferred participants are still in the conservative funds.

Figure 4: Share of participants in risky funds on all participants[6]



Source: IFP

Figure 5: Share of participants in risky funds on all participants and their savings' share in risky funds (%)



Source: IFP

The people who save in risky funds generally invest most of their savings into these funds. Additionally, the share in risky funds does not decrease significantly until the last 10 years before retirement, when individuals are obliged by law to allocate their savings into conservative assets. The level of share in risky assets is surprisingly high. About 65 % of all participants with savings in risky funds invest entirely into risky funds. The share of investment in stocks is around 70 %, indicating high risk tolerance of these savers.

BOX 1: Modelling probability of investing in risky funds

To determine which participants are most likely to invest in risky funds, we fit a model on II. pillar savings data from December 2017. The explained variable is an indicator whether individual saves at least partly in risky funds. It therefore only takes values 0 or 1. We use logistic regression, described by the following formula:

$$log\left(\frac{p}{1-p}\right) = X\beta + \varepsilon,$$

[4] In period from June 2009 to April 2012, PMC were obliged to produce non-negative return on savings, causing allocation in all funds entirely to bonds.
[5] Because we worked with anonymized data, we cannot know which fund was chosen by participant switching between PMCs.
[6] The comparison excludes one PMC (DSS Poštovej banky) for which historical data was unavailable.

where p is the probability of investing into risky assets, X is the set of explanatory variables and ε is the error term. The estimated coefficients with corresponding statistics are shown in Table 1.

Table 1: Results of logistic regression of probability of investing in risky funds.

	Coefficient	z-statistics	Pr>|z|	Odds ratio
Mandatory contribution (in €)	0.006	105.7	0	1.006
Voluntary contribution (dummy)	2.384	54.4	0	10.85
City (dummy)	0.222	42.6	0	1.249
Age above 19 years	-0.094	-326.2	0	0.911
AXA	1.422	133.7	0	4.146
ALLIANZ	1.035	102.4	0	2.816
NN	1.219	105.7	0	3.383
PB	0.816	65.9	0	2.263
VUBGEN	1.359	130.4	0	3.894
Constant	-0.522	-48.3	0	0.593

Individual is more likely to invest in risky funds if he is younger, earns more, makes extra contributions and lives in a city. For additional year of age, individual is 9 percent less likely to invest in risky assets and for every extra euro contributed, the probability increases by 0.6 percent. The choice of PMC also makes great effect, as the probability is 4 times as large in the most risky PMC as in the least. Surprisingly, gender or country of origin has no significant effect on the outcome. The model fit is however far from ideal. With the value of Pseudo R^2 of 0.12, most of the effects on the probability is random and it is quite difficult to generally determine the groups investing in risky assets, meaning that investing entirely in bonds is a feature more of a general population than certain groups.

3 Default strategies in pension saving

People are unwilling or unable to make decisions about their retirement saving. Recently, many countries besides Slovakia adopted defined contribution scheme instead of purely defined benefit scheme. Research shows that people face challenges when deciding on an optimal retirement saving strategy. This is due to limited financial literacy as well as behavioural and psychological biases. People face difficulties assessing risks related to pensions and long-term investment. Understanding compound interest rate, difference between real and nominal values and risk diversification cannot be taken for granted in general population. Additionally, people have present-biased time preferences, implying that short-term costs, such as singing up for II. pillar or choosing saving fund, can be a barrier to action in case of large delayed benefits, such as pensions (OECD, 2016a). People also avoid choice just to avoid making a decision that they would later regret, even if they are likely to benefit from them (OECD, 2018).

The share of investment in equities is among the smallest in international comparison (OECD, 2017). High share of investment in conservative assets results in one of the lowest real returns of pension assets in OECD (0.4 % in Slovakia, compared with OECD average of 3.7 %) between 2010 and 2015 (OECD 2016b). Introducing default strategy would increase share invested in equities and give people incentive to educate themselves about capital markets. Additionally, stock returns exhibit mean reversion, making stocks less volatile and therefore attractive in the long run.

BOX 2: Long term returns on stocks and bonds

Investing in stocks yielded on average more than two times larger real return than investing in bonds with 20 year investment period. We use data[7] from 1976 to 2018 and calculate returns on investment in stocks and bonds for various investment periods. The yields are calculated using the assumption that an individual contributes 100 euros every month. The return is than calculated as an average return of each 20 year period in this timeframe.

Table 2: Comparison of payoff mean and standard deviation for different saving horizons

Investment horizon	Contributions	Stocks investment	Bonds investment	Std. deviation stocks	Std. deviation bonds
1 month	100	100.6	100.3	4.22	1.61
1 year	1 200	1 249	1224	115.7	50.8
10 years	12 000	18 451	15 391	5 182	1 700
20 years	24 000	55 865	38 217	25 033	5 435

Investment in stocks yields higher return but is associated with higher risk. For shorter periods, the returns on investment to stocks and bonds do not differ much, whereas the standard deviation for stocks is significantly higher. For long intervals, the difference in returns from stocks and bonds increases rapidly.

Stocks tend to get safer in longer investment horizons. Probability of losing real value on investment in one month is about 40 % for both bonds and stocks. The average loss in stocks is 3.3 % compared to 1.1 % in bonds. If we extend investment horizon to 1 year, the probability of real loss drops to 29 % for stocks and 30 % for bonds. Although the probability of loss is larger in bonds, the expected loss is about three times larger for investment in stocks. In 20 years investment horizon, investment in bonds have never

[7] Using monthly data, for real stock returns was used MSCI US Total Return index and for real bond returns was used Barclays Aggregate US Total Return index, i.e. data are adjusted for inflation.

yielded negative return, for stocks this happened only in 0.3 % of cases. Long term investment therefore provides safer return on investment.

Introducing default saving strategy makes portfolio choice for passive participants. The default strategy should protect households form under-diversified, low-risk and low-return asset portfolios (Berardi, Tebaldi, Trojani, 2018). The introduction of default funds is also supported by international organizations (Rudolph, 2016; OECD, 2016a) and was advised by spending review[8] done by Value for money project in 2017.

Default saving strategy should take into account individual's life-cycle. Young person entering labour market possesses human capital, which resembles a large investment in inflation-indexed bond, with monthly payments in form of labour income. On the other hand, financial capital is limited at this stage. It therefore makes sense to invest in equities at young age and transfer parts of financial capital into conservative bond funds as individual ages and human capital becomes scarce. Additionally, because II. pillar pension will create only about one third of pensioner's income from labour savings, individual can choose more aggressive investment strategy, as the remaining pension paid from I. pillar resembles inflation-indexed annuity.

There is international consensus to use default lifecycling strategy. Optimal shifting strategy from risky to conservative assets varies across countries. Some countries define the maximum share of risky assets based on individual's age. Such format is currently used in Slovakia. Investors have to allocate 10 % of their assets in bond fund at age 52 and subsequently increase this share by 10 p.p. each year until age 61. If an individual actively requests so, the share of savings allocated to bond fund can only increase by 5 p.p. each year. By contrast in Sweden, if an individual does not choose a pension fund, he will be enrolled in AP7 fund managed by the state. His contributions will be then invested in stocks fund until the age of 55, after which 3-4 % of his assets are moved to a bond fund.

Pension saving funds can also be created by pooling savers by age cohorts to a target date fund. The share of stocks in this fund then varies as the age of participants increases, with a target retirement age. This is common practice for retirement saving in US, even though no default lifecycling strategy is set by the government. On the other hand in the UK, the government established the National Employment Savings Trust (NEST) with each person entering the labour market being automatically enrolled with an opt-out option. The default fund consist of three stages. In foundation stage, lasting about 5 years, the strategy is relatively conservative aiming to help younger members develop pension saving habit. Subsequently in growth phase, the portfolio becomes riskier with the target returns of 3 % over inflation level. This phase lasts approximately 30 years. For the last 10 years, the savings are transferred to conservative funds with aim to gradually reduce volatility near retirement age.

Alternatively, multiple funds with different allocations to equities can be created with individual switching funds based on one's age. In Chile, each PMC must create 4 funds (B, C, D and E) with creating aggressive fund A being optional, where each fund has a set maximum and minimum asset allocation to equities. New saver can choose a pension fund, with default option being B. As one ages, his savings are automatically transferred to fund C and D, making funds A and E purely for active savers.

Number of countries in CEE adopted default lifecycling strategies in pension saving. In Poland, portfolios consist of debt component, e.g. bonds or money market instruments and equity

[8] Spending review for labour market and social policies. Available in Slovak on http://www.finance.gov.sk/Default.aspx?CatID=11380

component, e.g. shares. For each five year age cohort, minimum and maximum share of equity component and debt component is defined. People aged 20-25, for example, must have at least 20 % invested in debt and 60 % in equity component. This ratio remains unchanged until age of 40, when it starts decreasing to at least 80 % in bonds and no minimum for equities (at age 60 and above). The exact share is then up to PMC to decide.

In Slovenia and Croatia, three funds with various exposure to risk are defined. Young individual in Slovenia begins pension saving in the dynamic fund and is shifted into calm and later guaranteed investment return fund. Pension fund manager defines the age, from which saver contributes to the more conservative fund, whereas the decision to transfer the savings is up to PMC, which has at most 3 years to do so in best interest of an individual. In Croatia, the proposal currently in parliamentary procedures consists of three funds based on their volatility and expected return. Individual is assigned into the most aggressive one for first 10 years. Subsequently, he is switched to medium fund, with choice to remain in aggressive fund for active savers. Last 10 years must then be spent in the conservative fund.

Academic literature is not in consensus about the optimal "one size fits all" default lifecycling strategy. There is a vast amount of literature addressing the problem of optimal lifecycling strategy. Manor (2017) compares lifecycling strategies against fixed ratio strategies. He finds that efficient strategies were the ones, where the share of stocks decreased stepwise or piecewise[9] as individual ages and the share of equities is higher for longer time and then decreases. Allocations with fixed ratio over time were found to be inefficient. Rudolph, Sabat (2016) find that aggressive[10] lifecycle strategy yields the best trade-off between expected return and risk. They also reparametrize the strategies, so that each one yields 70 % replacement rate. Even though the aggressive strategy remains optimal, authors note that the share of equity at retirement is relatively high (40 %) if the savings were to be annuitized.

Another approach accounts for sending new contributions to a more conservative fund or moving savings when an investment goal is achieved. Azoulay et al.; (2016) compared accumulating[11] strategy to usual switching strategy and found that accumulating strategy yields significantly higher return while the return in most adverse cases does not decrease critically. Basu et al.; (2009) used dynamic lifecycling to improve the efficiency of saving. After 20 years of saving in risky assets, the realized savings were compared against expected savings. If the goal was met, savings were switched to more conservative assets. If they again fall short of the investment goal, they are switched back to equities. They find that dynamic lifecycling earned less in the most adverse cases, however outperformed lifecycling in most (75-80 %) cases.

The optimal strategy depends on the type of benefit during the pay-out phase. Antolin et al.; (2010) compares fixed portfolio, lifecycling and dynamic investment strategies. The best strategies seemed to have constant equity share which dropped before retirement. They also found that the length of contribution period affects the ranking of strategies. With longer investment horizons, fixed ratio strategies seem to outperform lifecycling strategies. Most importantly, they show that optimal strategy depends on the pay-out phase.

[9] Stepwise strategy decreases allocation to equities in equally long „steps", whereas piecewise strategy allocates fixed ratio of savings to equities and then in the second part of investment period lowers this ratio.

[10] The aggressive strategy follows an allocation of 7 percent risk free assets up to age 40; 70 percent allocation to international equity; 20 percent to domestic equity; and 3 percent international fixed income. After age 40, the fund start gradually moving to allocations of 35 percent risk free assets, 39 percent international equity, 11 percent local equity, and 15 percent international fixed income at retirement age. (Rudolph, Sabat, 2016)

[11] Accumulating strategy consists of first allocating all savings to equity fund and then only sending new contributions to bond fund, while savings in equity fund accumulate.

Upon retirement, an individual can generally choose between buying life annuity, setting programmed withdrawal or cashing in a lump sum of savings. Life annuity provides protection against longevity. It however does not grant a choice in further investment nor provide flexibility in withdrawals in case of various contingencies (e.g. health expenses). Programmed and lump sum withdrawals give pensioner an opportunity to continue investing in a fund by their choice, however they do not protect from longevity. Antolin; (2009) recommends to provide pensioners with flexibility to allocate assets from DC fund if large part of their retirement income comes from a PAYG system. Additionally, he suggests setting a default strategy to buy deferred annuity at very old age (e.g. 85 years) already at retirement age and to give people full flexibility with savings until then.

4 Research methodology and data

To determine optimal default saving strategy, we simulate future returns on bonds and stocks. The investment horizon is set to 40 years, as it is approximately the length of pension saving. We simulate monthly returns on bonds and stocks based on historical data. The probability distribution of total savings at terminal point is then used to assess aptness of a strategy. We compare the expected savings with associated risk to find the optimal saving strategy.

We use three bootstrap simulation techniques to make projections about future returns on stocks and bonds. First, we use **naïve bootstrap**, where we assume that historical data are independently and identically distributed. We therefore resample historical returns with repetition, always using return on bonds and stocks in the same period. Second, to allow for autocorrelation either in mean or variance present in time series data, we simulate using non-overlapping **block bootstrap**. The idea is the same as with naïve bootstrap, however instead of resampling individual monthly observations, we resample blocks of observations. The block size is determined by rule of thumb, i.e. $b = \sqrt[3]{T}$, where T is number of observations in historical data. As we use 511 observations, the block size is set to 8.

Lastly, we use residual bootstrap based on parametric models estimated separately for bonds and stocks. Parametric approach fits ARMA-GARCH model on bond and stock returns (BOX 3). Future returns are then obtained using residual bootstrap. The advantage of this approach is that the behaviour of simulated time series better resembles the original data. The disadvantage is that individual processes are simulated independently.

BOX 3: Parametric model estimation and residual bootstrap

To preserve data generating process, we model returns using parametrical models and perform residual bootstrap. Returns on bonds are autocorrelated with its first lag and returns on stocks exhibit periods with high and low variance. To formalize the data generating process we fit model individually on bond and stock returns.

The model consists of mean model (ARMA) and variance model (GARCH). As there is no multiple period autocorrelation, we assume ARMA(1,1) model for mean:

$$\boldsymbol{y_t = \mu + \theta_1(y_{t-1} - \mu) + \theta_2\varepsilon_{t-1} + \varepsilon_t,}$$

where $\boldsymbol{y_t}$ is the original time series at time t, $\boldsymbol{\varepsilon_t}$ is the error term at time. For the variance model, we choose the usual approach in literature by assuming simple GARCH(1,1) model, i.e. the error term is given by the previous error term, previous variance and the innovation z_t:

$$\boldsymbol{\varepsilon_t = \sigma_t z_t \qquad z_t \sim N(0,1),}$$

$$\boldsymbol{\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\sigma_{t-1}^2.}$$

The residual bootstrap is then performed by resampling from estimated innovations $\boldsymbol{z_t}$. The method of resampling is none other than the one used with naïve bootstrap, with the exception that sampled innovation feeds as an input into parametrical model.

Both time series have statistically significant variance model, whereas the mean model is only used in case of bonds. For stock returns, constant mean is supposed. The mean model in case of bonds is only MA(1), supported by autocorrelation function with significant correlation only with the previous period. Models are fit on monthly returns using *rugarch* package in software R.

Table 3: Stocks model estimation

Parameter	Estimate	Standard error	P-value
μ	0.0069	0.0016	0.000
α_0	0.00005	0.00003	0.066
α_1	0.1123	0.2912	0.000
α_2	0.8654	0.0287	0.000

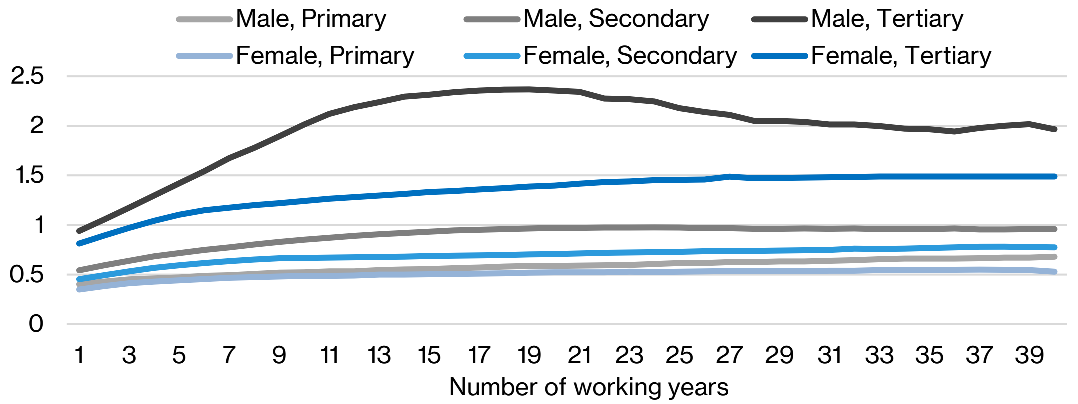
Table 4: Bonds model estimation

Parameter	Estimate	Standard error	P-value
μ	0.00257	0.0016	0.000
θ_2	0.2240	0.0471	0.000
α_0	0.000008	0.000003	0.023
α_1	0.1285	0.0208	0.000
α_2	0.8374	0.0271	0.000

Used data comprise of real returns on US stocks and bonds between January 1976 and August 2018. Using monthly data, for stock returns are represented by MSCI US Total Return index and for bond returns we used Barclays Aggregate US Total Return index. The chosen data consist of broad variety of assets and has the advantage of obtaining real returns by subtracting U.S. inflation from nominal returns. Average monthly bond return is 0.3 percent as opposed to 0.61 percent for stocks. Higher return on stocks is caused by additional volatility, as standard deviation in monthly stock returns is 0.042 compared to 0.016 for bonds.

Contributions to private pension savings were modelled based on estimated income profiles and current legislature. Six different income profiles were estimated on Slovak data from 2004 to 2017. Wages are assumed to increase according to the given wage profile and labour productivity growth rate[12]. Participant makes monthly contributions according to Slovak legislature. In 2019, the contribution is 4.75 % of one's gross wage. This share will increase by 0.25 p.p. until it reaches 6 % in 2024, where it will remain until investment horizon.

Figure 6: Income profiles for various pension savers (in proportion to average wage)



Source: IFP

[12] Labor productivity projections are used from Ageing Report 2018

Saver chooses the optimal investment every month based on a pre-defined saving strategy. The strategies stem from current literature. Three different sets of strategies are created: fixed strategies, lifecycling strategies and dynamic lifecycling strategies. A strategy can specify which part of contribution is allocated to which fund or at what time contributions should be sent to which fund. We assume that investor decides only between stocks and bonds.

BOX 4: Saving strategies

Fixed strategies – the ratio of savings in equity and bonds remains unchanged over the entire investment duration.

1. Equity- Savings are invested entirely into equity
2. Bonds- Savings are invested entirely into bonds
3. 60/40 – In each period, the share invested in equities is 60 % and in bonds 40%.

Lifecycling strategies – the ratio of funds in equity decreases with number of years to retirement based on a predefined rule.

- **Stepwise(start, end)**: The ratio decreases in 12 steps, every step represents the ratio of funds invested in equity. The strategy has two parameters, the starting ratio and the ending ratio. The tested strategies include:
    1. Stepwise(1, 0)
    2. Stepwise(1, 0.2)
    3. Stepwise(1, 0.2)
    4. Stepwise(0.8, 0.2)
    5. Stepwise(0.8, 0.4)
    6. Stepwise(0.6, 0.4)
- **Cont_stepwise(start, end):** Alternative to stepwise strategy, where contributions are distributed according to the given ratio. If the ratio is 80 percent, then this share of contribution is invested in stocks and rest in bonds. This makes it much more aggressive strategy compared to stepwise strategy.
    1. Cont_stepwise(1, 0)
    2. Cont_stepwise(0.8, 0)
- **Piecewise(start, end, years):** The allocation to equity is set to fixed ratio defined by parameter *start* for time given by parameter *years*. Then, the savings are allocated to bond fund in 10 steps to level given by parameter *end*.
    1. Piecewise(1, 0, 25)
    2. Piecewise(1, 0, 30)
    3. Piecewise(0.8, 0, 30)
    4. Piecewise(0.8, 0.2, 30)
    5. Piecewise(0.8, 0.4, 25)
- **Accumulation(start, years):** Individual begins by allocating funds between equity and bonds according to ratio *start*. After period given by parameter *years,* individual will invest contributions only into bonds.
    1. Accumulation(1, 20)
    2. Accumulation(1, 25)
    3. Accumulation(1, 30)
    4. Accumulation (0.8, 25)
    5. Accumulation(0.8, 30)

Dynamic lifecycling strategies – First 30 years, individual invests entirely into equities. After this period, the realized returns are each month compared against expected returns. If the realized return exceeds the expected one, savings are transferred into bonds based on strategy *Piecewise*(1, 0, 30). We define three different dynamic strategies:

1. Expected return- The goal is to generate yearly return rate of 3 % above inflation.
2. Decreasing return – After 30 years of saving, the return is compared with goal of 7.2 % per year. This goal decreases with each subsequent year to 1.8 % just before retiring.

3. Accumulation return – After 25 years of saving in equities, individual starts investing entirely into bonds. After 30 years of saving, he compares realized return on is savings in equities as in strategy *Expected return* and transfers savings when the goal is achieved.

Figure 7: Share of savings invested in stocks to all savings for chosen strategies (in %)

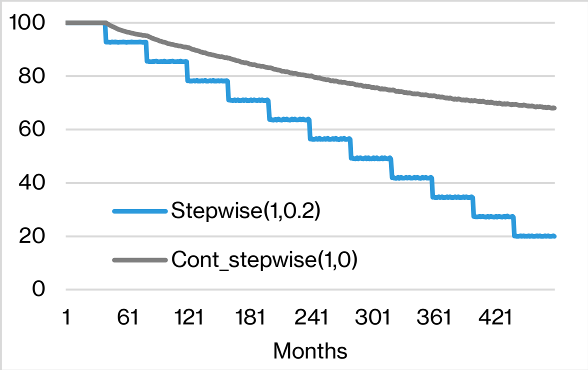
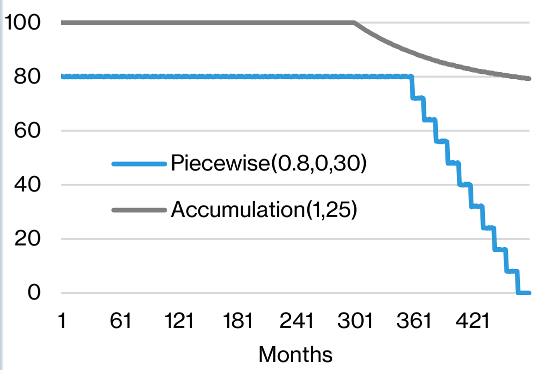


Figure 8: Share of savings invested in stocks to all savings for chosen strategies (in %)



The strategies are compared based on their expected return and associated risk. Profitability of a strategy is measured by average savings and median savings. The risk is measured by value at risk and conditional value at risk. Finally, Sortino ratio is used to obtain a single criterion for strategy comparison. The ratio uses a threshold value of savings and compares the additional return to associated risk. Average yield from bonds and non-negative real return are used as threshold.

BOX 5: Risk measures and Sortino ratio

1. **Value at risk (VaR)** is defined as α-th percentile of distribution function F_x of random variable X, i.e.

$$VaR_\alpha(X) = \min\{z|F_x(z) \geq \alpha\}, \alpha \in (0,1).$$

2. **Conditional Value at Risk (CVaR)** is defined as expected value of outcomes below α-th percentile, i.e.

$$CVaR(X) = E[X|X \leq VaR_\alpha(X)].$$

As opposed to unconditional value at risk, CVaR takes into account the distribution of outcomes below α –th percentile. If the distribution has a fat tail, this might be of bigger concern to a pension saver trying to omit very adverse outcomes.

3. **Sortino ratio** is defined as

$$S = \frac{R - T}{DR},$$

where R is the expected value of the distribution, T is the threshold savings level. Indicator DR defines the associated risk and is calculated as

$$DR = \sqrt{\int_{-\infty}^{T} (T - x)^2 f(x) dx},$$

where $f(x)$ is the probability density function at the saving's horizon.

5 Simulation results

Simulations for male with secondary education using block bootstrap simulation method is set as baseline. Income profile of such individual provides an appropriate representative agent. As a robustness check, simulations for all income profiles are run. Moreover, the results are corroborated using two other simulation methods (naïve bootstrap, residual bootstrap).

Accumulation strategies yield higher expected return, stepwise strategies are safer. The equity strategy has the highest expected value but is also the riskiest. Surprisingly, the bond strategy is not the safest, as conservative lifecycling strategies offer lower risk. This is because long term investing in bonds leads to lower appreciation of savings. Accumulation strategies are relatively more aggressive. The best performing parametrizations are the ones with allocation entirely into bonds during the first period of saving. On the other hand, stepwise strategies offer protection against very low savings. Stepwise strategies with higher allocations to equities have higher expected value, yet the overall level of risk is fairly similar across the whole class of stepwise strategies.

Table 5: Descriptive statistics for baseline simulation[13]

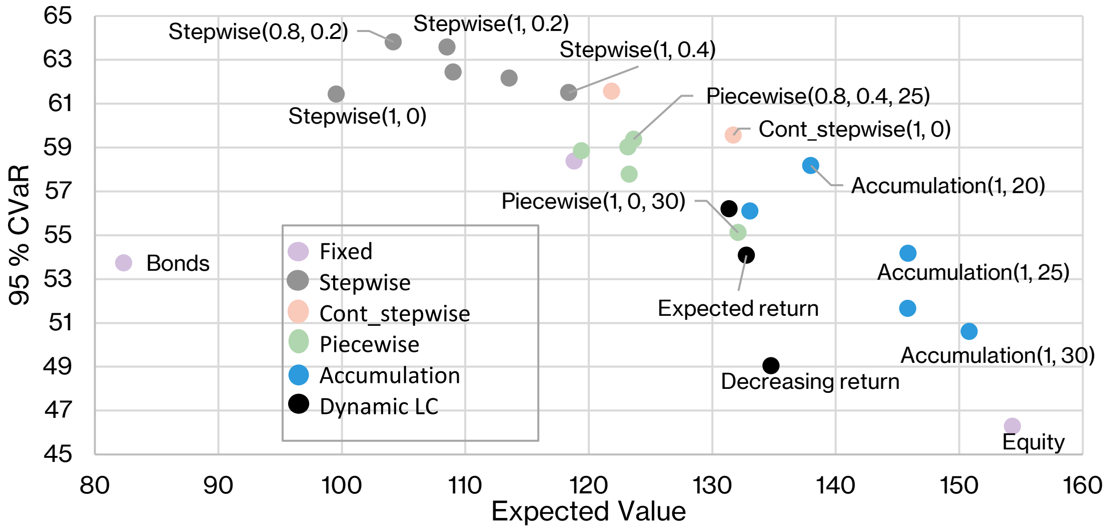
Strategy	Expected value	Median	95 % CVaR	95 % VaR
Bonds	82.3	80.4	53.7	58.3
Equity	154.3	131.5	46.3	57.3
60/40	118.8	111.6	58.4	65.9
Stepwise(1, 0)	99.6	96.1	61.4	67.4
Stepwise(1, 0.2)	108.5	103.6	63.6	69.9
Stepwise(1, 0.4)	118.4	111.5	61.5	68.5
Stepwise(0.8, 0.2)	104.2	100.6	63.8	69.8
Stepwise(0.8, 0.4)	113.6	107.9	62.2	68.8
Stepwise(0.6, 0.4)	109.0	104.2	62.4	69.0
Cont_Stepwise(1, 0)	131.7	114.4	59.6	66.2
Cont_Stepwise(0.8, 0)	121.8	108.4	61.6	67.8
Piecewise(1, 0, 25)	123.3	111.8	57.8	65.4
Piecewise(1, 0, 30)	132.1	118.0	55.1	62.9
Piecewise(0.8, 0, 30)	119.4	110.8	58.9	65.9
Piecewise(0.8, 0.2, 30)	123.2	114.4	59.0	66.5
Piecewise(0.8, 0.4, 25)	123.6	114.7	59.4	67.1
Accumulation(1, 20)	138.0	117.9	58.2	64.9
Accumulation(1, 25)	145.9	123.3	54.2	62.1
Accumulation(1, 30)	150.8	127.6	50.6	59.1
Accumulation(0.8, 25)	145.8	127.0	51.7	61.2
Accumulation(0.8, 30)	133.1	118.9	56.1	63.7
Expected return	132.8	118.2	54.1	66.5
Accumulation return	131.4	116.9	56.2	64.7
Decreasing return	134.8	122.5	49.0	64.3

Piecewise and dynamic strategies are almost always dominated by other strategies. Strategy dominates another one, if it offers higher expected return and lower risk. The primary criteria for comparison are expected value and CVaR. There is no dominant strategy in class Piecewise. Importantly, the lifecycling method currently used in Slovakia, strategy *Piecewise(1,0,30),* is clearly dominated by accumulation strategies. Dynamic lifecycling strategies also does not perform very well. While offering relatively high expected value, the associated risk is too high. The dynamic strategies are unable to prevent from the most

[13] Expected savings in 5 % worst scenarios (CVaR) were lower than sum of contributions only in case of equity strategy. CVaR is therefore displayed as expected savings in worst 5 % of scenarios and not the expected loss.

adverse outcomes. This is because for the worst 5 % of outcomes of every dynamic strategy, the individual never switches to more conservative saving.

Figure 9: Comparison of expected value and CVaR for saving strategies (in thousand euros)



The results are robust with respect to other evaluation criteria (median savings, value at risk). Performance evaluation based on expected value and CVaR might be biased due to outliers. Therefore, median (50th percentile) is compared to value at risk (5th percentile). Dynamic strategies show improvement as the share of outcomes which have never been switched to conservative strategies is around 5 %. However, the problem with a fat tail of the distribution prevents the strategy from being optimal.

Table 6: Comparison of simulation methods using Sortino ratio with various thresholds

Strategy	Naive, Bonds	Naive, Positive	Block, Bonds	Block, Positive	Residual, Bonds	Residual, Positive
Bonds	0.0	283	0.0	136	0.0	22
Equity	8.8	74	7.5	63	6.3	21
60/40	6.5	156	5.9	176	4.7	26
Stepwise(1, 0)	3.3	564	2.9	414	2.9	47
Stepwise(1, 0.2)	5.7	398	5.3	491	4.7	46
Stepwise(1, 0.4)	7.3	235	6.7	319	5.5	37
Stepwise(0.8, 0.2)	4.8	637	4.4	531	4.0	46
Stepwise(0.8, 0.4)	6.4	273	6.0	368	4.9	36
Stepwise(0.6, 0.4)	5.5	331	5.1	442	4.3	35
Cont_Stepwise(1, 0)	8.6	298	8.2	451	7.2	68
Cont_Stepwise(0.8, 0)	7.4	463	7.1	593	6.6	87
Piecewise(1, 0, 25)	6.9	156	6.2	198	5.5	37
Piecewise(1, 0, 30)	7.7	120	6.8	137	5.9	31
Piecewise(0.8, 0, 30)	6.6	157	5.9	193	5.1	34
Piecewise(0.8, 0.2, 30)	7.4	161	6.7	196	5.6	33
Piecewise(0.8, 0.4, 25)	7.6	175	7.0	222	5.7	33
Accumulation(1, 20)	9.3	269	8.6	392	7.5	65
Accumulation(1, 25)	9.4	156	8.4	198	7.2	39
Accumulation(1, 30)	9.2	107	8.1	105	6.8	28
Accumulation(0.8, 25)	9.0	105	7.9	93	6.5	32
Accumulation(0.8, 30)	8.0	147	7.3	187	7.0	33
Expected return	7.9	74	6.9	60	5.8	23
Accumulation return	8.0	141	7.2	173	6.1	35
Decreasing return	7.3	61	6.3	52	5.4	20

The optimal strategy compared to investing in bonds is *Accumulation(1, 20).* This strategy invests entirely in stocks during the first 20 years. Thereafter, the individual sends new contributions to a conservative fund. At retirement, the share of stocks in portfolio is around 70 percent, making it an aggressive strategy. Allocation to equities at the terminal point needs to be considered if the savings are annuitized or taken out as lump sum. For this reason, this strategy should only be used with appropriate withdrawal phase, which additionally decreases the share of stocks in portfolio. The strategy however remains optimal in regards to expected value to risk comparison.

As an alternative to optimal strategy, *Stepwise(1, 0.2)* provides for a safe choice. If savings are to be withdrawn or annuitized at retirement, the portfolio should already at this point be conservative to prevent adverse sharp declines. Using the zero real return criterion may not be adequate, as the chance of earning negative real return is extremely small for the safe strategies. This can be seen in high values of Sortino criterion as well as in inconsistency of best choice across simulation methods. The safest choices based on CVaR are the stepwise strategies with allocation share of 20 percent to equities at terminal point[14]. Of these two with similar risk profile, *Stepwise(1, 0.2)* provides significantly higher expected return and is thus chosen as the safe strategy.

Results are robust to changes in income profile, simulation method and length of career. Results do not change, when the simulation is performed for income profiles accounting for different gender and education level (Figure 6). Simulations using naïve bootstrap indicate *Accumulation(1, 25)* as the optimal strategy with *Accumulation(1, 20)* performing slightly worse. This is caused by not considering that price declines of equity usually last more than one period. For the safe strategy, the results based on Sortino are inconsistent. However, when considering the expected return to CVaR, the results favour *Stepwise(1, 0.2)* strategy. Finally, we consider the effect of increasing length of one's career. Simulations with career lasting 48 years[15] confirm baseline simulation results.

Future stock return uncertainty stems not only from unexpected return but also from imprecision of expected return. As emphasized by Pastor and Stambaugh (2012), the distribution of outcomes has two sources of dispersion, the deviation from expected return and the uncertainty in estimate of expected return. The former is the dispersion modelled by bootstrap. To check the robustness of results to the additional uncertainty, methodology from Fama, French (2017) is applied. Authors consider additional normally distributed dispersion term, with zero mean and standard deviation set to standard error of historical monthly returns. This dispersion term is then added to every monthly return for a given simulation run. This method is applied only to the baseline simulation, adjusting the method from naïve to block bootstrap method.

Results are robust to additional dispersion caused by uncertainty in expected return estimate. As expected, the variance of the distribution for return on equities increased due to the additional dispersion term. This is represented by decreased values for VaR and CVaR. The more risk averse strategies therefore perform slightly better, yet the baseline results remain valid.

Sensitivity tests on decreased future returns confirm robustness of results. Future returns on stocks and bonds may be lower than returns implied by historical data. This may be caused by demographic changes or decreased equity premium. Therefore, two robustness tests are run. Firstly, we consider lower stocks returns by 0.2 p.p. The mean of stock return distribution

[14] The 2 strategies are *Stepwise(0.8,0.2)* and *Stepwise(1,0.2)*
[15] In prolonged income profile, we assume that individual's wage increases at the same rate as average wage.

then shifts from 0.61 percent to 0.41 percent. Secondly, the distribution of monthly bond returns is shifted down by 0.1 p.p., with mean decreasing from 0.3 percent to 0.2 percent. The main results remain unchanged, even though in the later case, the strategy *Accumulation(1, 25)* performs slightly better. In case of the safe strategy, with only decreased stock returns, strategy *Stepwise(0.8, 0.2)* performs best. The performance of the chosen optimal and safe strategies is still very satisfactory.

6 Optimal strategy for the pay-out phase

The choice of saving strategy must account for the choice of pay-out phase. When an individual chooses to annuitize or to take a lump sum at retirement, one should have a smaller share in equities at that point than if programmed withdrawal was chosen. If a pensioner receives a large part of own overall pension from the I. pillar, one does not need to annuitize the savings at retirement. If savings continue to be invested, one can expect significantly higher level of pension. We consider a case where share in equities decreases for aggressive strategies during programmed withdrawal and savings are annuitized at higher age.

In programmed withdrawal, pensioner needs to choose investment strategy, number of years before annuitizing and a monthly withdrawal amount. Once the investor reaches retirement, one has a certain ratio of savings in equities. It is assumed that this ratio declines to 20 % at point of annuitizing. The programmed withdrawal is expected to last 10 years.

BOX 6: Life annuity calculation

We assume the following calculation of life annuity:

$$W = \sum_{i=1}^{t} p^j(i) * K * MC,$$

where W is the level of savings used to buy annuity, $p^j(i)$ is the probability of surviving month i after retirement for gender j. The parameter K denotes the monthly pension and MC is the mark-up used by insurance companies which considers the costs of annuity and associated risk. As the retirement age in Slovakia is steadily increasing due increasing life expectancy, it is assumed that the probability of surviving n months after retirement does not change in time.

Mark-up is calibrated based on data for annuitized savings in II. pillar. If an individual is to annuitize at retirement, this parameter is set to 0.8. If one is to do so after the programmed withdrawal, i.e. 10 years later, the mark-up is set to 0.7. Additionally, it is assumed that once the life annuity is bought, the monthly payments remain unchanged, i.e. there is no additional interest paid. Mark-up for indexed annuity offered in Slovakia is currently around 0.65. There is however expectation that with increasing sums being annuitized, better understanding of the industry, more participants and increased competition, this estimate might be pessimistic in the future.

Vast majority of pensioners would benefit from choosing programmed withdrawal opposed to annuitizing. It is assumed that pensioner chooses programmed withdrawal with monthly payments one would receive from buying an annuity at retirement. Had he bought an annuity after the programmed withdrawal, monthly payments would increase significantly, while associated risk would increase less than proportionally. For baseline simulation individual receives higher pension with probability of 93 percent, if one opts for programmed withdrawal.

Table 7: Expected value of annuities and CVaR at retirement (R) and after programmed withdrawal (PW) for chosen aggressive strategies (in euros)

Strategy	Annuity (R)	Annuity (PW)	CVaR (R)	CVaR (PW)
Cont_Stepwise(1,0)	534	744	241	192
Piecewise(0.8,0.4,25)	501	730	241	190
Accumulation(1,20)	559	767	236	193

Accumulation(1,25)	591	809	220	188
Accumulation(1,30)	611	836	205	182

Monthly payments after programmed withdrawal increase due to further investment and decreased life expectancy. Funds in programmed withdrawal are still invested during retirement, unlike life annuity payments (see BOX 6). Another reason for higher monthly payments is the rapidly decreasing probability of survival 10 years after retirement. As the number of participants decreases faster, the price of life annuity is significantly lower.

Prolonging dynamic lifecycling does not outperform accumulation strategy with pay-out phase. Dynamic lifecycling strategies could benefit from longer period to achieve investment return goal. Therefore, the investment period is extended to 50 years. The realized return is continuously being compared to the expected return starting 15 years before annuitizing. Life annuity at the terminal point can be then compared to the one with lifecycling strategy and pay-out phase. Table 8 shows, that prolonging of dynamic strategy does not improve its performance because it does not eliminate the most adverse outcomes.

Table 8: Expected value of annuities and CVaR after programmed withdrawal with I. pillar payments (in euros)

Strategy	Annuity	CVaR
Cont_Stepwise(1,0)	879	134
Piecewise(0.8,0.4,25)	799	126
Accumulation(1,20)	982	118
Accumulation(1,25)	1057	96
Accumulation(1,30)	1104	74
Expected return	925	69

Risk of inadequate pension persists with programmed withdrawal and subsequent annuitization. The risk of inadequate pension is not completely eliminated as the savings of pensioner at the end of programmed withdrawal might be insufficient to provide adequate pension. The solution might be a purchase of deferred annuity already at retirement. This annuity could be relatively cheap and a pensioner would have a guaranteed pension. This option is not modelled as no insurance company in Slovakia currently offers this product and it is difficult to accurately price it. If, however this choice was available, the risk of inadequate pension would be eliminated given that pensioner has sufficient funds at retirement.

7 Guarantees on return

Risk of insufficient funds at retirement can be partly mitigated by providing guarantees on return. Participants in default strategy could be saved from very adverse outcomes by contributing to a guarantee fund. This fund would be used to compensate those participants, affected by decrease in prices of equity shares. Part of every contribution to II. pillar would be sent to such fund.

BOX 7: Guarantee calculation

The probability distribution at terminal point of simulations is used to calculate the long term price of guarantees. Participants send a share of every contribution to a guarantee fund, which will be used to compensate people retiring with less than minimal return. It must therefore hold that

$$G = \sum_{i=1}^{k} p(W_i) * W_i, \qquad \forall\, W_i < W_{min}\,,$$

where W_i is the amount of savings for i-th trajectory of bootstrap and G denotes the amount of savings in the guarantee fund. Because guarantees account for very extreme case, instead of 5 000 simulations in baseline simulations, 25 000 simulations were done to obtain price of guarantees. The calculation does not consider any additional costs and assumes that savings in this fund retain their real value in time.

Guarantees could provide motivation to choose and remain in the default strategy. Importantly, this guarantee fund is intergenerational because if markets drop, all investors lose at the same time. All cohorts would contribute to such fund during their investment period, but only the cohort with very adverse outcome would earn a claim to this fund. The calculated price holds in long term, however in short term, savings in guarantee fund might be insufficient to cover all liabilities. This is because if market drops, multiple age cohorts are liable to lose their savings and claim resources from the guarantee fund.

The price of such guarantees depends on portfolio allocation. Up to this point, the nominal or the real value of returns was not of a primary concern, as different strategies were compared using the same assumptions. As long as the returns of stocks and bonds do not change dramatically, the results continue to hold for different investment choices. In case such guarantee fund existed, the rate of return would be important, too. Using the distribution of savings at terminal point, it is possible to calculate the amount saver needs to contribute.

It would cost 0.0002 percent of every contribution to guarantee a non-negative nominal return in the optimal strategy. Such scenario is extremely unlikely. If we were to guarantee non-negative real return, this would cost 0.04 percent of each contribution. In case guarantees on non-negative nominal return were introduced, the expected value of savings at retirement would decrease negligibly.

Sensitivity tests show, that even if returns decreased in the future, guarantees would not become expensive. We repeat simulations under assumptions of yearly returns decreased by 2 p.p. and 4 p.p. Only in case of real return guarantees under latest scenario would price of guarantees exceed 1 percent of each contribution in optimal strategy.

Table 9: Price of guarantees for optimal strategy (in % of II. pillar contribution)

Scenario	Nominal guarantees	Real guarantees
No guarantees	0	0
Baseline	0.0002	0.04
2 p.p. smaller returns	0.016	0.95
4 p.p. smaller returns	0.5	7.5

No trajectory in safe strategy ended up with savings less than sum of contributions. This means, that with safe strategy, there would possibly be no need for a guarantee fund. In case we would like to guarantee real return, the cost would be 0.006 percent of each contribution. Generally speaking, guarantees would be less expensive in safe strategy, because this strategy aims to eliminate such adverse outcome.

Table 10: Price of guarantees for safe strategy (in % of II. pillar contribution)

Scenario	Nominal guarantees	Real guarantees
No guarantees	0	0
Baseline	0	0.006
2 p.p. smaller returns	0.0033	0.57
4 p.p. smaller returns	0.335	7.55

Expected sum of savings at retirement does not significantly decrease with guarantees. In optimal and safe strategy, extra contribution to a guarantee fund in order to guarantee non-negative nominal return would have no effect on amount of savings. If non-negative nominal returns were guaranteed in optimal strategy with returns decreased by 4 p.p., this would decrease expected savings only by 0.5 %. Similarly, if non-negative real returns were guaranteed in optimal strategy with returns decreased by 2 p.p., expected savings would decrease by 1 %. Effect on amount of savings in the safe strategy would be even smaller.

8 Default strategy for current participants

Improvements to investment portfolio must also account for the current investors. If the portfolio choice for current investors does not improve, these future pensioners face high risk of inadequate pensions. 9 profiles of current participants investing only in the bond fund are assumed and modelled. Savings of participants are distributed between equities and bonds according to the aforementioned strategies[16]. Future returns are then simulated with shorter investment period.

Figure 10 : Switching current participants into default strategy

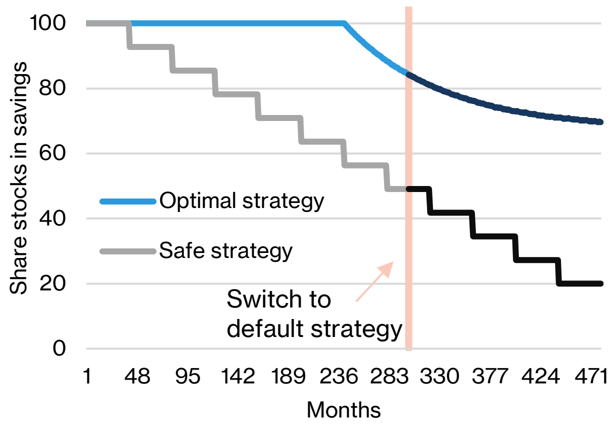
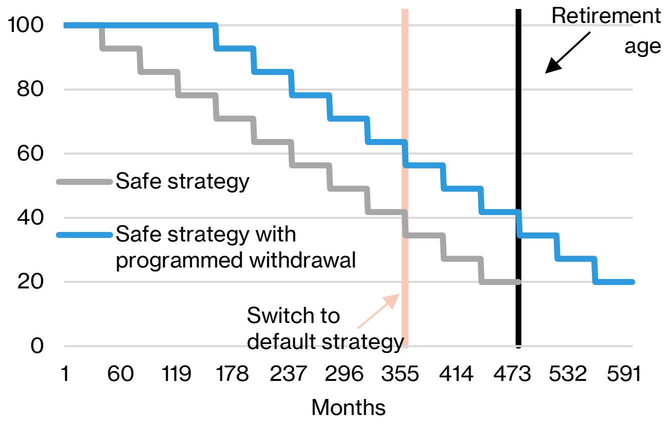


Figure 11 : Modification of safe strategy to consider pay-out phase



Modelled profiles consider that the sample of participants is very diverse due to frequent legislative changes. Three distinct investment durations were defined: 32, 22 and 12 years. Individual could also be participating for various lengths of time. If one earned average wage and participated for longer time, own savings should be approximately 7 000 €. For someone joining mid-way through, the saved amount is calibrated to 3 000 €. Alternatively, an individual with no savings is also modelled. These calibrations are then combined to create 9 unique profiles.

Current savers also have a choice to keep savings in conservative fund and invest only new contributions to stocks. Two extra strategies, applicable only for savers being switched to the default strategy are considered. Additionally, it is assumed that the individuals moved to a different strategy invest only in the bond fund. In the first strategy, the savings remain in the bond fund, however all new contributions are sent to the equity fund. The second strategy additionally takes into account the mandatory decrease in level of savings invested in equities defined by current legislature. These are named strategies *Contributions 1* and *Contributions 2.*

The best safe strategy remains *Stepwise(1, 0.2)*. Results do not change much with regards to length of investment period or initial savings. Additionally, the sooner the transfer is made, the more advantageous it would be. If participant has 3 000 € in his account, switching from bonds to the safe strategy might increase savings at retirement by 31 percent, assuming that one has 32 years to go, while only by 15 percent with 22 years to go. The risk measured by CVaR however also decreases from 17 percent in the former case to only 11 percent in latter.

[16] For example, if an individual was to choose safe optimal strategy with years of saving to go, 49 % of his savings would be allocated to equities and 51 % to bonds.

This also suggests that for people near retirement, switch would not have a significant impact on their savings.

Table 11: Descriptive statistics for participant with 7000 euros and 22 years of saving to go

Strategy	Expected value	Median	95 % CVaR	95 % VaR
Bonds	50.9	50.0	34.4	37.2
Equity	80.9	72.4	28.8	34.0
60/40	67.1	64.3	35.9	39.9
Stepwise(1, 0)	55.7	54.7	37.0	40.1
Stepwise(1, 0.2)	60.0	58.5	38.4	41.7
Stepwise(1, 0.4)	64.6	62.3	37.5	41.2
Stepwise(0.8, 0.2)	58.9	57.6	38.5	41.6
Stepwise(0.8, 0.4)	63.5	61.4	37.9	41.4
Stepwise(0.6, 0.4)	62.3	60.3	38.2	41.4
Cont_Stepwise(1, 0)	67.9	62.6	37.1	40.4
Cont_Stepwise(0.8, 0)	64.9	60.8	37.9	41.1
Piecewise(1, 0, 25)	64.8	61.8	35.9	39.7
Piecewise(1, 0, 30)	69.3	64.7	34.1	38.8
Piecewise(0.8, 0, 30)	65.1	62.1	36.0	40.0
Piecewise(0.8, 0.2, 30)	67.1	63.8	36.2	40.5
Piecewise(0.8, 0.4, 25)	67.4	64.1	36.5	40.7
Accumulation(1, 20)	73.0	68.3	33.6	38.4
Accumulation(1, 25)	76.8	70.5	31.6	36.7
Accumulation(1, 30)	79.3	71.7	30.1	35.2
Accumulation(0.8, 25)	76.6	70.0	32.0	36.8
Accumulation(0.8, 30)	72.8	68.1	33.3	38.0
Contributions 1	68.1	63.7	34.4	38.7
Contributions 2	62.0	59.6	36.8	40.2

Optimal strategy for current savers with bond strategy benchmark is accumulation strategy. The optimal number of years entirely in stocks increases as the investment period shortens. The strategy *Accumulation(1, 20)* is no longer optimal, but remains one of the most optimal strategies. Even though the ratio of return to risk is not optimal, the strategy offers the safest choice among class of optimal accumulation strategies.

The new strategies based on investing new contributions to equities do not perform well. Both contribution strategies are dominated when considering return to risk ratio. Additionally, these strategies work contrary to lifecycle concept. Share of savings in equities increases with age and is therefore too safe far from retirement and too risky before retirement. Contribution strategies perform well when an individual starts with no savings. This is no surprise as the strategy is then equivalent to equity strategy.

Participants near retirement can lose on switching to optimal strategy. Participants currently saving in conservative assets enjoy the advantage of low-volatility. If these resources were to be switched into the optimal strategy with high allocation to equities and market price would drop, they might go into retirement with less than if they remained in bonds. People near retirement should therefore be cautious about switching into optimal strategy.

Switching into the optimal strategy may be suboptimal for people with less than 12 years to retirement. We simulate how long it would take the savings in optimal strategy to perform at least as well as using the safe strategy in very adverse scenario. Such scenario is simulated by adding 10 % drop in value of savings in stocks at time of switch and another 20 % drop in value of stocks at time of retirement. Given that returns on stocks move independently of these shocks, it would take 12 years for individual to be indifferent between safe and optimal strategy.

For people with less than 12 years to retirement, the safe strategy could be prolonged into retirement. In case that pensioners are able to invest in stocks after retirement, they could hold 20 % of their savings in stocks, as the share would remained unchanged at the end of *Stepwise(1, 0.2)* strategy. Alternatively, they could use modified safe strategy, which makes use of the programmed withdrawal. The safe strategy with programmed withdrawal is the same as the *Stepwise(1, 0.2)* strategy, however participant would hold his savings in stocks for extra 10 years at the start of pension saving. The glide path is then shifted by ten years (Figure 11). Table 12 compares these two strategies and shows significant increase in expected value while the associated risk increases only mildly.

Table 12: Comparison of safe strategies with programmed withdrawal (in euros)

Strategy	Annuity after PW	CVaR
Safe strategy with constant 20 % in stocks during PW	160	72
Safe strategy with programmed withdrawal	175.4	70.5

Potential losses after switching to optimal strategy would probably be erased using programmed withdrawal. Stock returns exhibit mean-reversion, causing them to be higher after market falls. From October 2007 to February 2009, the stock index price dropped cumulatively by over 50 percent. The price returned to former maximum already in September 2012. If we compare investing separately in bonds and stocks on October 2007, the stock return would become more profitable by June 2014, i.e. already after 5 years and 4 months.

Optimal strategy for current participants would not be the same as for new investors. For new participants, it is intended that they send their contributions into equity fund for first 20 years and into bond fund for the rest. This means that in the optimal strategy there is no single path for share of equities in portfolio. Current contributors skip some part or the entire first phase of this strategy. They will therefore be switched into glide path modelled as average of all glide path simulated in baseline scenario (Figure 10).

To incentivize participants to switch to default strategy, guarantees on return can be offered. If savings for current participants are transferred into default strategy, not only future contributions but also the transferred savings need to be guaranteed. This is difficult for participants near retirement, because if stock prices fall right before their retirement, not only could they claim guarantees on new contributions but also on the saved amount, which would significantly increase the price of guarantees. However, there is currently only 36 000 participants in II. pillar with less than 5 years to retirement. Their average amount of savings is 7 000 euros. If they are switched into safe strategy with pay-out phase, their allocation to equities at time of retirement is approximately 35 %. The overall cost of claims in case of negative return could therefore be relatively small, compared to the increased expected return these participants would gain in programmed withdrawal.

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Appendix

Table A.1: Descriptive statistics for robustness check based on Fama, French

Strategy	Expected value	Median	95 % CVaR	95 % VaR
Bonds	82.1	80.2	53.4	57.7
Equity	175.7	130.7	35.5	45.4
60/40	123.4	111.2	48.9	57.5
Stepwise(1, 0)	100.5	95.1	57.3	62.6
Stepwise(1, 0.2)	111.0	103.0	55.9	62.2
Stepwise(1, 0.4)	123.4	111.0	51.5	59.3
Stepwise(0.8, 0.2)	105.7	99.8	57.2	63.2
Stepwise(0.8, 0.4)	117.0	107.6	53.0	60.5
Stepwise(0.6, 0.4)	111.3	104.2	54.4	61.7
Cont_Stepwise(1, 0)	148.4	114.6	52.1	58.5
Cont_Stepwise(0.8, 0)	135.1	108.3	55.4	61.1
Piecewise(1, 0, 25)	130.8	112.2	49.3	56.2
Piecewise(1, 0, 30)	142.9	117.7	45.2	52.5
Piecewise(0.8, 0, 30)	125.0	110.2	50.0	56.7
Piecewise(0.8, 0.2, 30)	129.8	113.1	48.9	56.5
Piecewise(0.8, 0.4, 25)	130.3	113.9	49.0	57.3
Accumulation(1, 20)	157.3	117.8	50.5	56.3
Accumulation(1, 25)	166.6	123.4	45.2	52.4
Accumulation(1, 30)	172.1	127.1	40.4	48.5
Accumulation(0.8, 25)	162.8	127.2	40.1	49.6
Accumulation(0.8, 30)	144.7	117.9	46.4	53.8
Expected return	143.2	117.8	37.7	48.9
Accumulation return	142.0	117.1	45.7	53.1
Decreasing return	144.6	121.8	35.7	45.7

Table A.2: Descriptive statistics for robustness check with lower stock returns

Strategy	Expected value	Median	95 % CVaR	95 % VaR	Sortino bond	Sortino real
Bonds	82.1	80.1	53.7	57.7	0.00	153.0
Equity	95.8	84.0	32.3	38.4	0.71	11.9
60/40	90.2	85.2	46.1	51.7	0.62	39.1
Stepwise(1, 0)	85.9	83.3	54.3	58.5	0.37	157.4
Stepwise(1, 0.2)	87.8	84.5	52.9	57.9	0.55	128.2
Stepwise(1, 0.4)	89.8	85.2	48.8	53.7	0.64	59.5
Stepwise(0.8, 0.2)	87.1	84.1	54.3	58.9	0.50	174.8
Stepwise(0.8, 0.4)	89.0	85.1	50.3	55.4	0.61	75.3
Stepwise(0.6, 0.4)	88.2	85.0	51.8	56.6	0.57	96.5
Cont_Stepwise(1, 0)	91.0	82.8	49.5	53.9	0.71	91.9
Cont_Stepwise(0.8, 0)	89.3	83.1	52.5	56.6	0.64	165.9
Piecewise(1, 0, 25)	90.3	83.8	45.7	50.8	0.59	39.8
Piecewise(1, 0, 30)	91.8	83.4	41.7	47.1	0.63	25.0
Piecewise(0.8, 0, 30)	89.8	84.4	46.3	51.7	0.59	42.3
Piecewise(0.8, 0.2, 30)	90.6	84.8	45.6	51.3	0.64	38.9
Piecewise(0.8, 0.4, 25)	90.8	85.1	46.0	51.7	0.66	40.4
Accumulation(1, 20)	91.9	82.3	47.8	52.1	0.72	67.1
Accumulation(1, 25)	93.6	82.5	42.5	47.2	0.72	30.3
Accumulation(1, 30)	94.7	83.1	37.6	43.1	0.71	17.8
Accumulation(0.8, 25)	94.4	84.5	37.0	43.2	0.71	16.6
Accumulation(0.8, 30)	92.1	84.1	43.4	48.6	0.68	31.7
Expected return	92.4	85.8	33.4	40.5	0.61	12.2

Accumulation return	91.8	83.8	42.7	47.7	0.64	29.7
Decreasing return	93.1	87.7	32.4	38.6	0.61	11.4

Table A.3: Descriptive statistics for robustness check with lower stock and bond returns

Strategy	Expected value	Median	95 % CVaR	95 % VaR	Sortino bond	Sortino real
Bonds	66.8	65.6	44.2	47.4	0.00	16.9
Equity	96.3	84.0	32.4	38.3	2.65	12.1
60/40	82.9	78.9	42.5	47.6	2.16	22.0
Stepwise(1, 0)	74.1	71.8	46.6	50.5	1.12	32.7
Stepwise(1, 0.2)	78.1	75.3	47.1	51.2	1.83	39.5
Stepwise(1, 0.4)	82.2	78.3	44.8	49.0	2.29	30.3
Stepwise(0.8, 0.2)	76.4	73.9	47.7	51.7	1.60	41.6
Stepwise(0.8, 0.4)	80.4	77.3	45.6	50.0	2.10	32.1
Stepwise(0.6, 0.4)	78.7	76.0	46.3	50.6	1.88	33.4
Cont_Stepwise(1, 0)	85.9	77.4	44.6	48.3	2.64	35.0
Cont_Stepwise(0.8, 0)	82.1	75.6	46.3	50.2	2.30	41.1
Piecewise(1, 0, 25)	83.9	77.9	42.1	46.7	2.16	22.5
Piecewise(1, 0, 30)	87.4	79.0	39.5	44.6	2.35	18.2
Piecewise(0.8, 0, 30)	82.6	77.5	42.7	47.4	2.08	23.3
Piecewise(0.8, 0.2, 30)	84.2	78.4	42.6	47.3	2.32	23.5
Piecewise(0.8, 0.4, 25)	84.5	79.2	42.9	47.5	2.41	24.6
Accumulation(1, 20)	87.9	77.8	43.6	47.1	2.75	32.0
Accumulation(1, 25)	91.7	80.0	40.0	44.2	2.82	21.5
Accumulation(1, 30)	94.2	81.5	36.5	41.0	2.76	15.9
Accumulation(0.8, 25)	93.0	82.5	36.3	41.5	2.71	15.2
Accumulation(0.8, 30)	87.8	79.9	40.8	45.3	2.53	21.3
Expected return	88.7	81.8	33.2	40.0	2.23	11.1
Accumulation return	87.4	79.8	40.2	44.5	2.42	19.7
Decreasing return	90.3	84.9	32.5	38.4	2.21	10.7

Table A.4: Sortino ratio with bond threshold for current savers (investment period, savings)

Strategy	32, 7000	32, 3000	32, 0	22, 7000	22, 3000	22, 0	12, 7000	12, 3000	12, 0
Bonds	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Equity	8.2	6.7	6.2	4.8	4.1	3.7	2.3	2.1	1.8
60/40	6.3	5.3	4.8	4.0	3.5	3.1	2.0	1.9	1.6
Stepwise(1, 0)	2.9	2.5	1.9	1.1	0.9	0.7	0.3	0.2	0.2
Stepwise(1, 0.2)	5.0	4.5	3.7	2.6	2.3	2.0	1.1	1.0	0.8
Stepwise(1, 0.4)	6.6	5.7	5.0	3.8	3.3	2.9	1.8	1.7	1.4
Stepwise(0.8, 0.2)	4.3	3.8	3.1	2.2	2.0	1.7	1.0	0.9	0.8
Stepwise(0.8, 0.4)	6.0	5.2	4.5	3.5	3.1	2.8	1.7	1.6	1.4
Stepwise(0.6, 0.4)	5.3	4.6	4.0	3.2	2.9	2.6	1.6	1.5	1.3
Cont_Stepwise(1, 0)	8.3	7.0	5.7	4.3	3.6	2.5	2.0	1.4	0.6
Cont_Stepwise(0.8, 0)	7.4	6.2	4.7	3.8	3.0	2.0	1.7	1.2	0.5
Piecewise(1, 0, 25)	6.1	5.3	4.7	3.2	2.8	2.4	1.0	0.9	0.7
Piecewise(1, 0, 30)	6.8	5.8	5.3	3.9	3.3	2.9	1.6	1.4	1.1
Piecewise(0.8, 0, 30)	6.0	5.1	4.5	3.4	2.9	2.5	1.3	1.1	0.9
Piecewise(0.8, 0.2, 30)	6.7	5.7	5.2	4.0	3.5	3.1	1.8	1.6	1.4
Piecewise(0.8, 0.4, 25)	7.1	6.0	5.5	4.2	3.7	3.3	1.9	1.8	1.5
Accumulation(1, 20)	8.1	6.8	6.3	4.7	4.0	3.6	2.2	2.0	1.7
Accumulation(1, 25)	8.3	6.8	6.4	4.9	4.2	3.7	2.3	2.1	1.8
Accumulation(1, 30)	8.3	6.8	6.3	4.9	4.2	3.8	2.4	2.2	1.8
Accumulation(0.8, 25)	8.3	6.9	6.4	4.9	4.3	3.8	2.3	2.2	1.8

Accumulation(0.8, 30)	7.5	6.2	5.8	4.5	3.9	3.5	2.2	2.1	1.7
Contributions 1	4.4	4.9	6.2	2.8	3.1	3.7	1.2	1.5	1.8
Contributions 2	3.2	3.4	4.2	1.8	1.9	2.1	0.6	0.7	0.6

Table A.5: Descriptive statistics for participant with 7000 euros, 12 years of saving to go and shock 10 % today and 20 % at date of retirement

Strategy	Expected value	Median	95 % CVaR	95 % VaR
Bonds	30.5	30.2	22.9	24.3
Equity	30.0	28.5	14.3	16.3
60/40	30.4	29.7	19.4	21.1
Stepwise(1, 0)	30.9	30.6	23.4	24.7
Stepwise(1, 0.2)	30.8	30.5	23.2	24.4
Stepwise(1, 0.4)	30.7	30.2	21.5	23.1
Stepwise(0.8, 0.2)	30.8	30.4	23.2	24.4
Stepwise(0.8, 0.4)	30.6	30.2	21.7	23.2
Stepwise(0.6, 0.4)	30.6	30.1	21.8	23.2
Cont_Stepwise(1, 0)	30.8	30.0	22.3	23.6
Cont_Stepwise(0.8, 0)	30.7	30.1	22.8	24.0
Piecewise(1, 0, 25)	31.7	31.3	23.3	24.7
Piecewise(1, 0, 30)	32.6	31.9	22.3	23.8
Piecewise(0.8, 0, 30)	32.1	31.6	23.0	24.4
Piecewise(0.8, 0.2, 30)	31.7	31.1	22.1	23.7
Piecewise(0.8, 0.4, 25)	30.9	30.4	21.1	22.6
Accumulation(1, 20)	30.4	29.5	18.0	19.8
Accumulation(1, 25)	30.4	29.3	16.6	18.5
Accumulation(1, 30)	30.4	29.0	15.5	17.5
Accumulation(0.8, 25)	30.8	29.7	17.5	19.3
Accumulation(0.8, 30)	30.5	29.5	17.6	19.5
Contributions 1	29.8	29.0	19.6	21.0
Contributions 2	32.6	32.1	23.9	25.3

All results are available in an excel file published with this analysis.